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Maximizing Reliability of Two-Commodity Flow Networks with Time Constraint Using Fuzzy Optimization

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Abstract

The main objective of this paper is to study the optimal components assignment problem for a two-commodity flow network via disjoint minimal paths in the existence of system reliability and total lead-time bands. By using fuzzy membership functions, we formulated the problem as fuzzy linear optimization. An algorithm based on genetic algorithm with fuzzy optimization is suggested for solving the presented problem. The proposed approach achieved success in finding an optimal solution that characterized with maximum reliability and minimum total lead-time. Three studied cases are given four nodes, six nodes and fourteen nodes computer networks to illustrate the efficiency of our proposed approach for solving the presented problem.

Keywords

Network Reliability, Stochastic-Flow Networks, Components Assignment Problem, Fuzzy Multi-Objective Linear Programming, Two-Commodity, Genetic Algorithms.

1. Introduction

Lin et al.[1] give a definition to the network reliability over stochastic-flow network SFN as the probability that SFN able to transmit particular amount of flow from source to sink effectively. Components assignment problem (CAP) is crucial for the research on system reliability analyses and it is one main problem in this field to search on the optimal component assignment for maximizing the system reliability and make the system performance get better as demonstrated [2]. A lot of researchers researched on CAP for a SFN in the existence of several constraints using several algorithms to maximize the network reliability. Lin [3] evaluated the system reliability states in the existence of demand, time and budget constraints then he presented an algorithm in order to find every minimal system in terms of such system states.

Lin and Yeh [4] concentrated on getting the optimal carrier selection in the presence of budget constraint relied on network reliability standard. Lin and Yeh [5] researched on the multi-state CAP and proposed an optimization method relied on GA for maximizing the network reliability in the

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existence of budget constrain. Lin and Yeh [6] discussed multi-objective optimization for stochastic computer networks to maximize network reliability and minimize total coast, where they proposed an approach to solve such cases. Lin and Yeh [7] discussed the multi-objective CAP in the presence of assignment cost and reliability for SFN, and they presented two-stage approach for solving such a problem. Hamdy et al. [8] presented an approach relied on GA with fuzzy optimization for solving optimal CAP in a SFN in which each arc and node has several possible capacities and lead-time. Hassan [9] proposed a GA to get optimal components while studying CAP for SFN in the existence of lead-time constraint to minimize total lead time and maximize the system reliability. Hassan and Abdou [10] worked on the multi-objective CAP in the presence of lead-time constraint and they proposed a GA rely on the Non-dominated Sorting Genetic Algorithm II (NSGA-II) to find optimal components that minimizing total lead time and maximizing the network reliability. Aissou et al.[11] studied the CAP for SFN in which every component has jointly an assignment cost and lead -time bands, they proposed approach relied on a random weighted GA to maximize the network reliability, minimize total lead–time and minimizing cost. Hamdy et al. [12] presented an approach relied on GA with fuzzy optimization for solving optimal CAP for maximizing network reliability and minimizing lead time and cost.

Many researchers searched on system reliability in flow network for multicommodity [13-24]. Lin [13] proposed an algorithm in terms of minimal paths to generate each lower boundary points in a SFN for a two-commodity demand in which each node and arc have many of capacities and may fail. Hassan and Abdou [14] discussed the reliability evaluation under different conditions, in one of them they studied transmitting multicommodity demand through disjoint path and they proposed an algorithm to evaluate the network reliability for this case. Lin [15] proposed an algorithm to generate each lower boundary points in a SFN of multicommodity demand in the existence of budget constraint to estimate system reliability, in which each node and each arc owns much of possible capacities.

Bellman and Zadeh [25] presented the basics of decision making in the fuzzy environments. Zimmerman [26] explained that Fuzzy Linear Programming (FLP) had the ability to resolve the linear vector maximum issues. Luhandjula [27] presented an overview at core ideas that made the burgeoning body of fuzzy mathematical programming affirming the methodological view. Wang [28] gathered the fundamentals of multi-objective programming application by utilizing a membership function of the linear expression to exemplify and combine each fuzzy objective. Tang and wang [29] introduced an inexact approach and a GA for obtaining a group of inexact solutions with agreeable membership degree for solving objective and resource kind of FLP issues. In addition to this, a lot of researchers talked about FLP problem formulations and membership functions as illustrated [30 – 44] to use FLP on various problems and improved the obtaining solutions.

Newly, FLP is applied to solve several problems [45-50]. Mutingi [46] employ a fuzzy multi-objective GA in obtaining high quality solutions to resolve the multi-objective decision issue. Chang et al. [47] employ a fuzzy multi-objective linear programming method to combine a new character with the current components by using an optimization method of the highest match. Dinagar and Kamalanathan [48] presented a proposition about a new ranking techniques of Subinterval average and subinterval addition in order to resolve FLP issue. Sinha et al. [49] presented a FLP manner for solving an issue in food industry. Gue et al.[50] applied a FLP to find the optimal design to the tri generation method (the power generation, generation of cooling effect and heat generation) .

The main objective of this paper is to solve the CAP for a two-commodity flow network via disjoint minimal paths provided that system reliability and total lead time are existence as constrain. An approach based on GA is proposed to solve the presented problem. This study is different from [8] which discussed CAP in a SFN with node failure in which each arc and node has several possible capacities and lead-time. And different from [12] which discussed CAP in a SFN for maximizing network reliability and minimizing lead time and cost.
The paper is systematic as follows: Section 2 clarifying necessary notations. Section 3 displays the problem formulation. After, section 4 explaining the fuzzy linear formulation to the current problem. Section 5 illustrating the proposed multi-objective GA based on FLP. Section 6 demonstrating studied cases. The latest section shows conclusion.

2. Notations

N No. of nodes.

v $\{a_e | 1 \leq e \leq v\}$: No. of arcs.

MPs Disjoint minimal paths.

np Number of minimal paths.

mp_j Minimal path no. j; $j = 1, 2, \dots, np$.

L_j The lead time of mp_j.

vc The number of available components.

d_i The amount of commodity $i, i = 1,2$.

vn_k The components number $k, k = 1,2, \dots, vc$.

$l(vn_k)$ Lead time of components vn_k.

$R_{d_1,d_2;T}$ The system reliability to the given demand d_1, d_2 under time limit T, for simplicity using R.

x Capacity vector defined as $\mathcal{X} = (x_1, x_2, \dots \dots, x_e)$.

P $(p_1, p_2, \dots, p_v)$ the assigned components in which vn_k is assigned to a_e.

$S_l(P)$ Total lead time.

b_j^i The bandwidth used by commodity d_i on mp_j .

w_i The consumed capacity on a_e by commodity $d_i, i = 1, 2$.

B_j The upper bound for the bandwidth of mp_j.

R_{obj} Maximum acceptable feasible values of R .

R_0 Minimum acceptable feasible values of R .

S_l^{obj} Minimum acceptable feasible values of $S_l(P)$.

S_l^0 Maximum acceptable feasible values of $S_l(P)$.

$\mu(S_l)$ Fuzzy objective membership functions of $S_l(P)$.

μ(R) Fuzzy objective membership functions of R .

α The acceptable membership degree level.

3. The Reliability Evaluation

Let $c(a_e)$ be the maximum capacity on a_e by d_i, then B_j, [14] and [16], is given by

$$B_j = \min_{a_e \in mp_j} \{c(a_e)\} \quad (1)$$

where $c(a_e)$ the maximum capacity on a_e by d_i .

and $b_j^i \leq B_j$ for each bandwidth b_j^i which evaluated by the following equation:

$$b_j^i = \frac{\max_{a_e \in mp_j} \{w_i(a_e)\} \times d_i}{T - L_j} \quad (2)$$

Finally, each $\mathcal{X}_e$ in the capacity vector X is constructed by:

$$\mathcal{X}_e = \sum_{j=1}^{np} \sum_{i=1}^{u} \rho_j b_j^i \quad \text{where } \rho_j = \begin{cases} 1, if\ a_e \in mp_j \\ 0, if\ a_e \notin mp_j \end{cases} \quad (3)$$

Then the system reliability is evaluated as follow:

$$R = Pr\{\cup_{i=1}^{q} \{Y | Y \geq X^q\}\},\ Pr\{Y\} = Pr\{y_1\}.Pr\{y_2\} \ldots .Pr\{y_v\}. \quad (4)$$

Where q is number of obtained vectors X.

The following algorithm is used to evaluate the reliability

Begin

For each arc read $c(a_e)$.

Read $d_i, T, and\ L_j$.

For j=1 to np do

Calculate B_j and b_j^i according to equation (1) and (2) respectively.

End do

Determine the set of disjoint paths.

Construct the capacity vector $\mathcal{X}$ using equation (3).

Evaluate R using equation (4).

End

4. Problem formulation

The mathematical programming formulations of the multi-objective optimization problem to maximize system reliability of a flow network and minimize total lead-time are demonstrating as follow:

$$\text{Maximize } R_{d_1,d_2;T}(P) \quad (5)$$

$$\text{Minimize } S_l(P) \quad (6)$$

Subject to:

$$p_e = k, k \in \{1,2, \ldots, vc\} \text{ for e} = 1,2, \ldots, \text{v.} \quad (7)$$

$$p_e \neq p_h \text{for } e \neq h \tag{8}$$
$$L_j \leq T, \quad \text{j} = 1,2, \dots, \text{np} \tag{9}$$

Where:

$$L_j = \sum_{e=1}^{V} l(p_e) \Big|_{p_e \in mp_j} \tag{10}$$
$$S_l(P) = \sum_{e=1}^{V} l(p_e) \tag{11}$$

Bands (7) and (8) confirm that every link must be given one component and that every component can be assigned to at most one link. All feasible component assignments are producing using bands (7) and (8). Constraint (9) ensures that the lead-time of the path MP_j (L_j) is less than the time limit (T), [9].

5. Fuzzy linear Formulation

To transform the mathematical formulation showed in the problem formulation portion in to fuzzy linear formulation [51], We will realize that R_{obj} and S_l^{obj} are the objective values where $R \leq R_{obj}$
and $S_l(P) \geq S_l^{obj}$ [12] , to get the optimal solution the next two equations are used in our research.

$$\mu(R) = \begin{cases} 1 & \text{if} \quad R > R_{obj} \\ 1 - \dfrac{R_{obj} - R}{\mathcal{P}_0} & if \quad R_{obj} - \mathcal{P}_0 \leq R \leq R_{obj} \\ 0 & \text{if} \quad R < R_0 \end{cases} \tag{12}$$

And

$$\mu(S_l) = \begin{cases} 1 & if \quad S_l(P) < S_l^{obj} \\ 1 - \dfrac{S_l(P) - S_l^{obj}}{\mathcal{P}_1} & if \quad S_l^{obj} \leq S_l(P) \leq S_l^{obj} + \mathcal{P}_1 \\ 0 & if \quad S_l(P) > S_l^0 \end{cases} \tag{13}$$

Where:

$\mathcal{P}_0$ Tolerance of $\mu(S_l)$, $\mathcal{P}_0 = R_{obj} - R_0$.

$\mathcal{P}_1$ Tolerance of $\mu(R), \mathcal{P}_1 = S_l^0 - S_l^{obj}$.

So, the membership function of the decision space $\widetilde{S}$ is $\mu_{\tilde{S}}(P)$is calculated with:

$$\text{Max } \mu_{\tilde{S}}(P) = \text{Max}\{0, \min\{\mu(\text{R}), \mu(S_l)\}\} \tag{14}$$

6. The Genetic Algorithm

6.1 Chromosome Modeling

The chromosome P is represented by the following equation:

$$P = (p_1, p_2, \dots, p_v) \tag{15}$$

Where $p_1, p_2, \dots, and\ p_v$ are random component numbers between 1 and vc, this mean that the component p_1 is assigned to an arc a_1, the component p_2 is assigned to arc $a_2, \dots$, and the component p_v is assigned to arc a_v.

6.2 Initial population

The following steps addresses the initial population:

Step1: generate P chromosome randomly in the form:

$$P = (p_1, p_2, \dots, p_v).$$

Step 2: calculate $R\ and\ S_l(P)$.

Step 3: Obtain $\mu_{\tilde{S}}(P)$ based on eq. 0.

Step 4: if $\mu_{\tilde{S}}(P)$ of P in step 1 is lower than α ignore it and back to step1.

Step 5: repeat step 1 to 3 to generate $\mathcal{S}$chromosomes.

6.3 The fitness function

Fuzzy accurate solution of the membership $\mu_{\tilde{S}}(P)$ is considered as fitness function F of GA.

6.4 Genetic Selection

The selection process based on roulette wheel selection method according to the following steps:
Step 1: obtain $pr(gn), gn = 1,2, \dots, \mathcal{S}$ by:

$$pr(gn) = \frac{\mu_{\tilde{S}}(P)}{\sum_{gn=1}^{\mathcal{S}} \mu_{\tilde{S}}(P) + \varepsilon} \tag{16}$$

Whereε is small positive integer.

Step 2: initiate random real number r in [0, 1].

Step 3: $\leq pr(1)$, take the first chromosome, otherwise take the gn_{th} chromosome
$(2 \leq gn \leq \mathcal{S})\ if\ pr(gn-1) < r \leq pr(gn)$.
Step 4: Repeat steps 2 and 3, $\mathcal{S}$ times and find $\mathcal{S}$ chromosomes.

6.5 Genetic crossover operation

In the proposed GA, uniform crossover is used to breed a child from two as shown in figure1.The crossover operation is performed as follows:
Step 1: chose two chromosome related to the selection strategy, section 6.4.

Step 2: randomly choose a component from one of the two chromosomes to form a corresponding components of the child.

Step3: repeat step 2 until the components of the child fill up perfectly.

Parents		Offspring		The final offspring
(5, 2, 1, 3, 4, 6)				
	$\Longrightarrow$	(5, 1 $\rightarrow$ 3, 1,2,4,5 $\rightarrow$ 6)	$\Longrightarrow$	(5,3,1,2,4,6)
(3, 1, 4, 2, 6, 5)				

Figure 1. Uniform crossover operator

6.6 Genetic mutation operation

A child undergoes mutation based on the mutation probability g_m and the g_m can be obtained based on the following steps.

Step 1: generate a random number $r_1 \in [0,1]$.

Step 2: if $r_1 < g_m$, the chromosome is chosen to mutate and go to step 3, otherwise skip this chromosome.

Step 3: for each component of the child do:

Step 3.1: Generate a random number $r_2 \in [0,1]$.

Step 3.2: if $r_2 < g_m$ then mutate this component as follows:

Step 3.2.1: if $p_j = vn_k$, then randomly choose one in $\{1,2,\dots,vc\} - \{vn_k\}$.

Step 3.2.2:if previous step does not achieves skip this component.

Figure 2 shows an example of performing the mutation operation on a given chromosome.

Parents $\Longrightarrow$ offspring $\Longrightarrow$ The final offspring

(5,8,1,3,4,6) $\Longrightarrow$ ($\underline{5} \rightarrow$ 8,3,1,$\underline{2} \rightarrow$ 2,4,6) $\Longrightarrow$ (8,3,1,2,4,6)

Figure 2. Mutation operation

6.7 The whole optimization approach

For solving such problem, the following steps addressing the implementation of algorithm:

Begin

Input $\mathcal{S}, \mathscr{g}, g_m, g_c, S_l^{obj}, S_l^0, R_{obj}, R_0, d_1, d_2$ and α .

Generate the initial population.

Calculate $\mu(R)$ and $\mu(S_l)$ for each chromosome P using equation (12) and (13).

Calculate $\mu_{\tilde{s}}(P)$and $pr(gn)$ using equation (14) and (16).

While $(gn \le \mathscr{g})$do

$k = 1$

While $(k \le \mathcal{S})$ do

Generate new chromosome P by applying GA operators; g_c and g_m.

Calculate $\mu_{\tilde{s}}(P)$.

If $\mu_{\tilde{s}}(P) \ge \alpha$ then $k = k + 1$.

End do

Save best solution with highest $\mu_{\tilde{s}}(P)$.

$gn = gn + 1$.

End do

End

7. Studied Cases:

This portion introduces the results of stratifying the suggested approach on three networks, four nodes, six nodes and fourteen nodes. The genetic parameters used in the proposed GA are: $\mathcal{S} = 10, \mathscr{g} = 100, g_c = 0.95, g_m = 0.05, 0.3 \le \alpha \le 0.6$.

7.1 Four Nodes Network Example

The network clarifying in figure 3 [9] has four nodes and six arcs. The capacity, probability, lead-time and weights of every component (vn) is illustrating in Table 1 [9]. There are two disjoint paths: $mp_1 = \{a_1, a_5\}, mp_2 = \{a_2, a_6\}$.

We studied various values for T under various values of α where $R_0 = 0.9, R_{obj} = 0.99, S_l^0 = 15, S_l^{obj} = 11$. Figure 4 clarifies the $best\ \mu_{\tilde{s}}(P)$ for various values of (d_1, d_2, T). Furthermore tables 2, 3, 4, 5 and 6 demonstrates the values for $\alpha, best\ \mu_{\tilde{s}}(P), R\ , S_l(p)\ and\ P$.

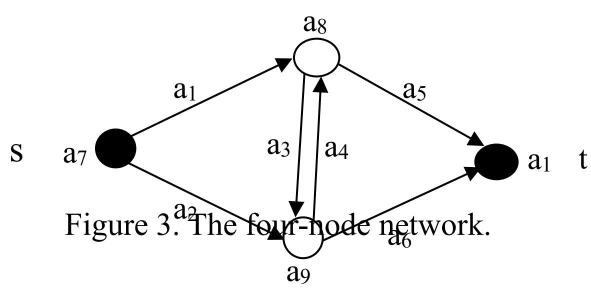
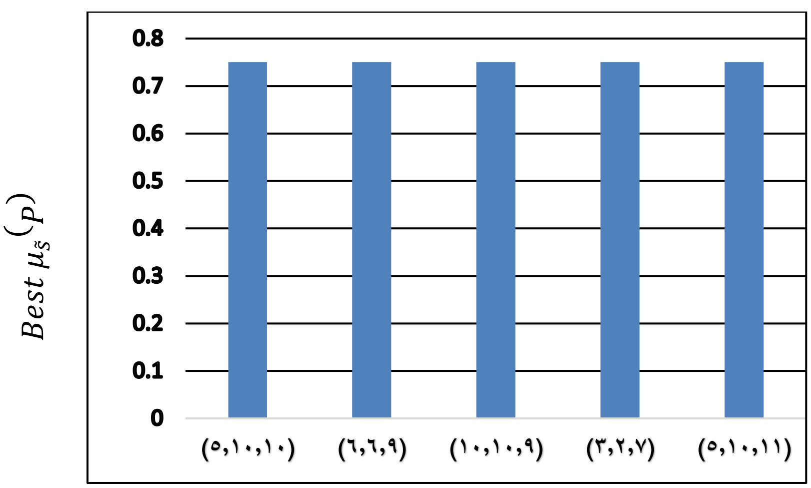


Figure 3. The four-node network.

Table 1. Components capacities, probabilities, lead-time, and consumed capacities.

vn_k	**Capacity**							$l(vn_k)$	w_1	w_2
	0	1	2	3	4	5	6			
1	0.01	0.00	0.01	0.00	0.01	0.00	0.97	2	١	٢
2	0.05	0.05	0.05	0.15	0.20	0.50	0	3	١	٢
3	0.07	0.08	0.00	0.85	0	0	0	2	١	٢
4	0.70	0.00	0.00	0.00	0.00	0.30	0	2	١	٢
5	0.01	0.00	0.00	0.05	0.00	0.00	0.94	1	١	٢
6	0.01	0.00	0.00	0.01	0.00	0.00	0.98	3	١	٢
7	0.50	0.50	0	0	0	0	0	3	١	٢
8	0.25	0.25	0.50	0	0	0	0	1	١	٢
9	0.15	0.25	0.10	0.10	0.10	0.10	0.20	2	١	٢
10	0.00	0.05	0.05	0.90	0	0	0	2	1	٢



(d_1, d_2, T)

Figure 4. The $best\ \mu_{\tilde{s}}(P)$ for different values of (d_1, d_2, T).

Table 2. Optimal solutions for the network in figure 3, when T=10.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(5,10),10	0.3	0.500	0.999005	13	2 1 4 6 10 5
	0.4	0.500	0.999405	13	1 2 4 6 10 5
	0.5	0.750	0.992786	12	5 8 4 2 6 9
	0.6	0.750	0.999207	12	10 3 4 5 1 6

Table 3. Optimal solutions for the network in figure 3, when T=9.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(6,6),9	0.3	0.500	0.999405	13	1 2 4 6 10 5
	0.4	0.500	0.999405	13	1 2 6 4 10 5
	0.5	0.750	0.992786	12	5 8 4 2 6 9
	0.6	0.750	0.999207	12	10 3 4 6 1 5

Table 4. Optimal solutions for the network in figure 3, when T=9.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(10,10),9	0.3	0.500	0.984679	13	1 2 7 6 8 5
	0.4	0.500	0.999405	13	1 2 4 6 10 5
	0.5	0.750	0.978431	12	2 8 4 5 6 9
	0.6	0.750	0.999207	12	10 3 4 5 1 6

Table 5. Optimal solutions for the network in figure 3, when T=7.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(3,2),7	0.3	0.500	0.999405	13	1 10 4 6 2 5
	0.4	0.500	0.999405	13	1 2 4 6 10 5
	0.5	0.750	0.992786	12	5 8 4 2 6 9
	0.6	0.750	0.999207	12	10 3 4 5 1 6

Table 6. Optimal solutions for the network in figure 3, when T=11.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(5,10),11	0.3	0.500	0.999405	13	1 2 4 6 10 5
	0.4	0.500	0.999405	13	5 2 4 6 10 1
	0.5	0.750	0.996169	12	5 2 4 8 6 9
	0.6	0.750	0.999207	12	10 3 4 5 1 6

7.2 Six Nodes Network Example

The network has six nodes and nine arcs as illustrated in figure 5 [10]. The table contains arc capacity, probability, lead-time and weights are clarifying in table 7[10].The two disjoints MPs are as follow: $mp_1 = \{a_1, a_4, a_9\}, mp_2 = \{a_2, a_6, a_8\}$.

We studied various values for T under various values for α where $\mathrm{R}_0 = 0.9, \mathrm{R}_{\mathrm{obj}} = 0.99$, $S_l^{obj} = 15, S_l^0 = 20$. Figure 6 clarifies the $best\ \mu_{\tilde{s}}(P)$ for various values of(d_1, d_2, T). Furthermore tables 8, 9, 10, 11, 12 and 13 shows the values for $\alpha, best\ \mu_{\tilde{s}}(P), R\ , S_l(p)\ and\ P$.

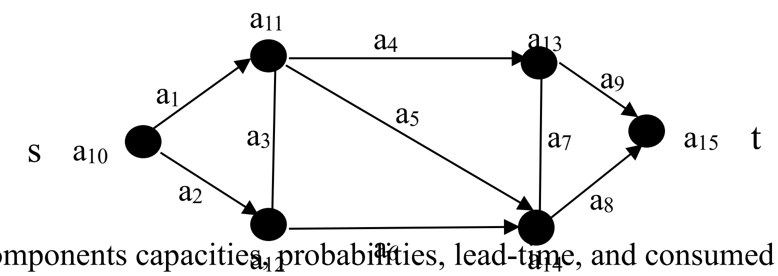


Figure 5. The six-node network.

Table 7. Components capacities, probabilities, lead-time, and consumed capacities.

vn_k	Capacity							$l(vn_k)$	w_1	w_2
	0	1		3	4	5	6			
1	0.01	0.00	0.01	0.00	0.01	0.00	0.97	2	1	2
2	0.05	0.05	0.05	0.15	0.20	0.50	0	3	1	2
3	0.07	0.08	0.00	0.85	0	0	0	2	1	2
4	0.70	0.00	0.00	0.00	0.00	0.30	0	2	1	2
5	0.01	0.00	0.00	0.05	0.00	0.00	0.94	1	1	2
6	0.01	0.00	0.00	0.01	0.00	0.00	0.98	3	1	2
7	0.50	0.50	0	0	0	0	0	3	1	2
8	0.25	0.25	0.50	0	0	0	0	1	1	2
9	0.15	0.25	0.10	0.10	0.10	0.10	0.20	2	1	2
10	0.00	0.05	0.05	0.90	0	0	0	2	1	2
11	0.01	0.99	0	0	0	0	0	1	1	2
12	0.02	0.00	0.05	0.00	0.05	0.00	0.88	1	1	2
13	0.07	0.00	0.28	0.00	0.00	0.65	0	3	1	2
14	0.05	0.05	0.90	0	0	0	0	2	1	2
15	0.60	0.40	0	0	0	0	0	2	1	2
16	0.15	0.00	0.00	0.00	0.85	0	0	1	1	2
17	0.10	0.10	0.10	0.70	0	0	0	1	1	2
18	0.70	0.00	0.00	0.00	0.00	0.30	0	3	1	2
19	0.07	0.18	0.75	0	0	0	0	2	1	2
`20	0.40	0.40	0.20	0	0	0	0	3	1	2

Table 8.Optimal solutions for the network in fig.5, when T=11.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(10,10),11	0.3	0.991	0.989206	14	17 12 10 14 1 11 4 5 9
	0.4	0.900	0.980998	14	17 14 10 11 1 12 4 5 9

	0.5	0.800	0.990864	16	14 10 1 17 7 11 8 2 5
	0.6	0.800	0.995900	16	14 10 17 1 7 11 8 2 5

Table 9.Optimal solutions for the network in figure 5, when T=11.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(5,10),11	0.3	0.991	0.989206	14	17 11 10 14 1 12 4 5 9
	0.4	0.991	0.989206	14	17 11 10 14 1 5 4 12 9
	0.5	0.800	0.995900	16	14 10 17 1 7 11 8 2 5
	0.6	0.800	0.995900	16	14 2 17 1 7 11 8 10 5

Table 10.Optimal solutions for the network in figure 5, when T=11.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(10,5),11	0.3	0.800	0.997330	16	3 11 15 14 18 10 4 5 12
	0.4	0.991	0.989206	14	17 11 10 14 1 12 4 5 9
	0.5	0.800	0.997330	16	3 11 15 14 18 5 4 10 12
	0.6	0.800	0.995900	16	14 10 17 1 7 11 8 2 5

Table 11.Optimal solutions for the network in figure 5, when T=9.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(9,6),9	0.3	0.900	0.980998	14	17 11 10 5 1 12 4 14 9
	0.4	0.991	0.989206	14	9 11 10 14 1 12 4 5 17
	0.5	0.800	0.995900	16	14 1 17 10 7 11 8 2 5
	0.6	0.800	0.995900	16	14 10 17 1 7 11 8 2 5

Table 12.Optimal solutions for the network in figure 5, when T=9

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P$	R	$S_l(p)$	P
(9,9),9	0.3	0.991	0.989206	14	17 11 10 9 1 12 4 5 14
	0.4	0.991	0.989206	14	17 11 10 14 1 12 4 5 9
	0.5	0.800	0.997880	16	14 10 17 2 7 11 8 1 5
	0.6	0.800	0.997880	16	10 14 9 1 19 11 16 2 5

Table 13.Optimal solutions for the network in figure 5, when T=10.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(10,10),10	0.3	0.812	0.973078	14	17 9 10 14 1 12 4 5 11
	0.4	0.991	0.989206	14	17 11 10 14 1 12 4 5 9
	0.5	0.800	0.995900	16	14 10 17 1 7 11 8 2 5
	0.6	0.800	0.995900	16	14 10 17 1 7 11 8 2 5

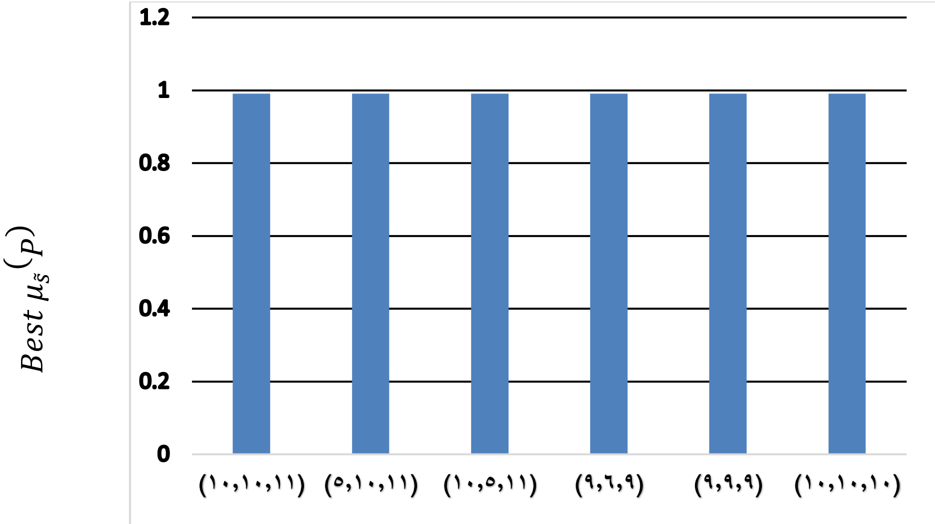


Figure 6. The $best$ [illegible] ues of (d_1, d_2, T).

7.3 Fourteen Nodes Network Example

The network clarified in figure 7[52] has 14 nodes and 22 arcs. The capacity, probability lead-time and weights of every component (vn) is clarifying in Table 14 [52] [53]. There are two disjoint minimal paths: $mp_1 = \{a_1, a_2, a_3\}, mp_2 = \{a_4, a_5, a_6\}$.

Under various values for α we studied various values for d_1, d_2 and T .Where $R_0 = 0.9, R_{obj} = 0.99, S_l^{obj} = 50, S_l^0 = 65$. Figure 8 clarifies the $best\ \mu_{\tilde{s}}(P)$ for various values of (d_1, d_2, T). Furthermore tables 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27 and 28 shows the values for $\alpha, best\ \mu_{\tilde{s}}(P), R\ , S_l(p)\ and\ P$.

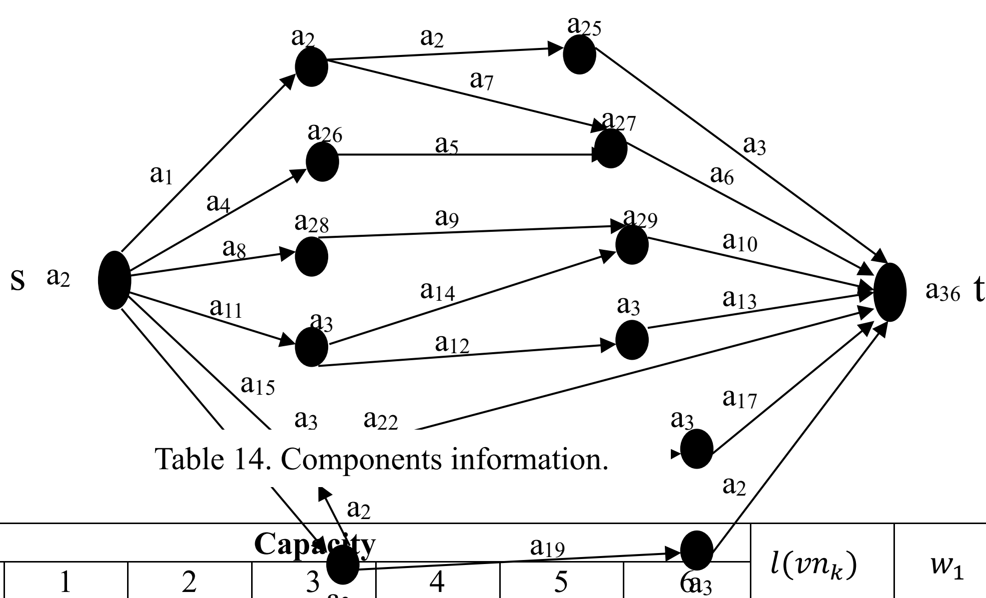


Figure 7. The fourteen-node network.

Table 14. Components information.

vn_k	Capacity							$l(vn_k)$	w_1	w_2
	0	1	2	3	4	5	6			
1	0.05	0.05	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	2
2	0.05	0.05	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	2
3	0.05	0.05	0.05	0	0.85	0	0	3	1	2
4	0.05	0.05	0.0	0.05	0.0	0.85	0	3	1	2
5	0.05	0.05	0.0	0.10	0.0	0.80	0	4	1	2

6	0.05	0.05	0.05	0.0	0.85	0	0	3	1	2
7	0.10	0.0	0.90	0	0	0	0	2	1	2
8	0.05	0.05	0.0	0.05	0.0	0.85	0	3	1	2
9	0.05	0.05	0.05	0.0	0.85	0	0	4	1	2
10	0.05	0.05	0.10	0.0	0.80	0	0	2	1	2
11	0.05	0.05	0.0	0.05	0.0	0.85	0	3	1	2
12	0.05	0.05	0.05	0.0	0.05	0.0	0.80	2	1	2
13	0.05	0.05	0.05	0.0	0.10	0.0	0.75	1	1	2
14	0.05	0.0	0.95	0	0	0	0	2	1	2
15	0.05	0.05	0.0	0.05	0.0	0.05	0.80	3	1	2
16	0.05	0.05	0.0	0.05	0.10	0.0	0.75	2	1	2
17	0.05	0.05	0.0	0.05	0.0	0.85	0	3	1	2
18	0.05	0.05	0.0	0.05	0.85	0	0	3	1	2
19	0.05	0.05	0.0	0.05	0.0	0.85	0	3	1	2
`20	0.05	0.05	0.05	0.0	0.85	0	0	3	1	2
21	0.05	0.0	0.95	0	0	0	0	2	1	2
22	0.05	0.95	0	0	0	0	0	4	1	2
23	0.06	0.03	0.0	0.05	0.0	0.86	0	2	1	2
24	0.05	0.02	0.0	0.03	0.0	0.90	0	2	1	2
25	0.04	0.04	0.04	0.0	0.88	0	0	3	1	2
26	0.05	0.05	0.0	0.05	0.0	0.85	0	3	1	2
27	0.05	0.05	0.0	0.05	0.0	0.85	0	4	1	2
28	0.04	0.04	0.04	0.0	0.88	0	0	3	1	2
29	0.04	0.0	0.96	0	0	0	0	4	1	2
`30	0.05	0.05	0.0	0.05	0.0	0.85	0	3	1	2

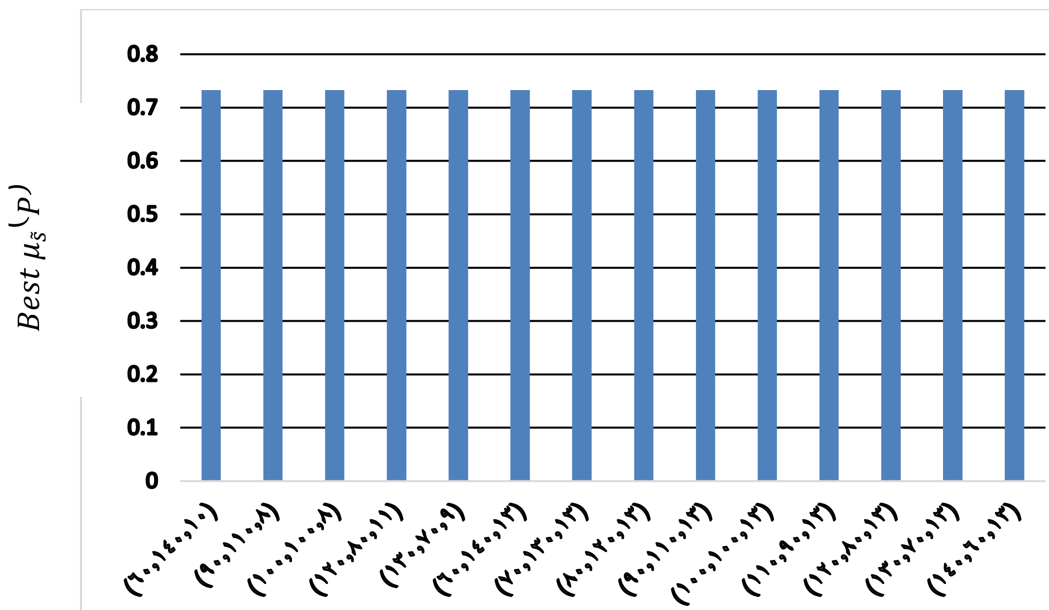


Figure 8. The $best\ \mu_{\tilde{s}}(P)$ for different values of (d_1, d_2, T).

Table 13. Optimal solutions for the network in figure 7, when T=13.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{S}}(P)$	R	$S_l(p)$	P
(60,140),13	0.3	0.667	0.979658	55	17 2 24 21 30 3 28 11 10 23 13 12 6 29 20 26 1 25 8 16 14 7
	0.4	0.600	0.982246	56	28 3 29 24 21 10 25 16 12 7 2 26 11 1 14 20 4 18 6 8 19 13
	0.5	0.733	0.980945	54	1 3 16 18 12 25 7 10 2 6 24 21 8 23 13 20 28 17 4 15 26 14
	0.6	0.667	0.982246	55	15 1 13 3 25 28 4 10 20 30 14 21 2 17 27 6 12 24 18 7 23 16

Table 16. Optimal solutions for the network in figure 7, when T=13.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{S}}(P)$	R	$S_l(p)$	P
(70,130),13	0.3	0.667	0.979658	55	17 12 24 21 30 3 28 11 10 23 13 7 6 29 20 26 1 25 8 16 14 2
	0.4	0.600	0.982246	56	28 3 29 24 21 10 25 16 12 7 2 26 11 1 14 20 4 18 6 8 19 13
	0.5	0.733	0.980945	54	1 3 16 18 12 25 7 10 2 6 24 21 8 23 13 20 28 17 4 15 26 14
	0.6	0.667	0.982246	55	25 1 28 14 15 8 3 2 11 30 12 21 13 17 23 10 6 24 27 7 20 16

Table 17.Optimal solutions for the network in figure 7, when T=13.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{S}}(P)$	R	$S_l(p)$	P
(80,120),13	0.3	0.667	0.979658	55	17 2 24 21 30 3 28 11 10 23 13 12 6 29 20 26 1 25 8 16 14 7
	0.4	0.600	0.982246	56	28 3 29 24 21 10 25

					16 12 7 2 26 11 1 14 20 4 18 6 8 19 13
	0.5	0.733	0.980945	54	1 3 16 18 12 25 7 10 2 6 24 21 8 23 13 20 28 17 4 15 26 14
	0.6	0.667	0.982246	55	15 1 13 3 25 28 4 10 20 30 14 21 2 17 27 6 12 24 18 7 23 16

Table 18. Optimal solutions for the network in figure 7, when T=13.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(90,110),13	0.3	0.667	0.979658	55	17 2 24 21 30 3 28 11 10 23 13 12 6 29 20 26 1 25 8 16 14 7
	0.4	0.600	0.981123	56	23 3 24 2 25 29 6 16 12 7 10 15 17 1 26 20 21 30 18 8 4 13
	0.5	0.733	0.980945	54	1 3 16 18 12 25 7 10 2 6 24 21 8 23 13 20 28 17 4 15 26 14
	0.6	0.667	0.982246	55	15 1 13 3 25 28 4 10 20 30 14 21 2 17 27 6 12 24 18 7 23 16

Table 19. Optimal solutions for the network in figure 7, when T=13.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(100,100),13	0.3	0.667	0.979658	55	17 2 24 21 30 3 28 11 10 23 13 12 6 29 20 26 1 25 8 16 14 7
	0.4	0.600	0.981123	56	23 3 24 2 25 29 6 16 12 7 10 15 17 1 26 20 21 30 18 8 4 13
	0.5	0.733	0.980945	54	1 3 16 18 12 25 7 10 2 6 24 21 8 23 13 20 28 17 4 15 26 14
	0.6	0.667	0.982246	55	25 1 28 14 15 8 3 10

					11 30 12 21 13 17 23 2 6 24 27 7 20 16

Table 20. Optimal solutions for the network in figure 7, when T=13.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(110,90),13	0.3	0.667	0.979658	55	17 12 24 21 30 3 28 11 10 23 13 7 6 29 20 26 1 25 8 16 14 2
	0.4	0.600	0.982246	56	28 3 29 24 21 10 25 16 12 7 2 26 11 1 14 20 4 18 6 8 19 13
	0.5	0.733	0.980945	54	1 3 16 18 12 25 7 10 2 6 24 21 8 23 13 20 28 17 4 15 26 14
	0.6	0.667	0.982246	55	25 1 28 14 15 8 3 2 11 30 12 21 13 17 23 10 6 24 27 7 20 16

Table 21. Optimal solutions for the network in figure 7, when T=13.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(120,80),13	0.3	0.667	0.979658	55	17 2 24 21 30 3 28 11 10 23 13 12 6 29 20 26 1 25 8 16 14 7
	0.4	0.600	0.982246	56	28 3 29 24 21 10 25 16 12 7 2 26 11 1 14 20 4 18 6 8 19 13
	0.5	0.733	0.980945	54	1 3 16 18 12 25 7 10 2 6 24 21 8 23 13 20 28 17 4 15 26 14
	0.6	0.667	0.982246	55	15 1 13 3 25 28 4 10 20 30 14 21 2 17 27 6 12 24 18 7 23 16

Table 22. Optimal solutions for the network in figure 7, when T=13.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(130,70),13	0.3	0.667	0.979658	55	17 2 24 21 30 3 28 11 10 23 13 12 6 29 20 26 1 25 8 16 14 7
	0.4	0.600	0.982246	56	28 3 29 24 21 10 25 16 12 7 2 26 11 1 14 20 4 18 6 8 19 13
	0.5	0.733	0.980945	54	1 3 16 18 12 25 7 10 2 6 24 21 8 23 13 20 28 17 4 15 26 14
	0.6	0.667	0.982246	55	15 1 13 3 25 28 4 10 20 30 14 21 2 17 27 6 12 24 18 7 23 16

Table 23. Optimal solutions for the network in figure 7, when T=13.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(140,60),13	0.3	0.600	0.980945	56	17 16 2 28 30 11 12 23 10 4 22 13 26 3 20 25 1 24 8 18 14 21
	0.4	0.600	0.981123	56	23 3 24 2 25 29 6 16 12 7 10 15 17 1 26 20 21 18 30 8 4 13
	0.5	0.733	0.980945	54	1 3 16 18 12 25 7 10 2 6 24 21 8 23 13 20 28 17 4 15 26 14
	0.6	0.667	0.982246	55	15 1 13 3 25 28 4 10 20 30 14 21 2 17 27 6 12 24 18 7 23 16

Table 24. Optimal solutions for the network in figure 7, when T=10.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(60,140),10	0.3	0.667	0.979658	55	17 1 24 21 30 3 28 11

					10 23 13 12 6 29 20 26 7 25 8 16 14 2
	0.4	0.600	0.980945	56	28 3 19 24 21 10 25 16 12 7 2 26 11 1 14 20 4 18 6 8 29 13
	0.5	0.667	0.980945	55	18 12 25 24 6 10 23 4 13 26 30 14 1 16 7 22 2 21 28 20 3 19
	0.6	0.733	0.979658	54	19 14 8 20 21 30 15 16 17 13 24 7 2 28 23 12 6 10 11 1 3 4

Table 25. Optimal solutions for the network in figure 7, when T=8.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{S}}(P)$	R	$S_l(p)$	P
(90,110),8	0.3	0.667	0.973222	55	14 1 7 13 24 4 20 11 16 28 21 30 10 2 8 25 22 26 15 5 19 12
	0.4	0.667	0.979658	55	21 14 8 20 12 16 13 2 27 24 26 7 17 3 23 6 30 10 19 1 18 4
	0.5	0.733	0.980945	54	1 2 28 14 21 8 13 7 6 10 3 11 17 12 16 23 30 24 19 26 18 4
	0.6	0.733	0.980945	54	2 24 28 1 23 17 26 21 19 6 12 16 10 4 7 8 25 3 14 13 30 18

Table 26. Optimal solutions for the network in figure 7, when T=8.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{S}}(P)$	R	$S_l(p)$	P
(100,100),8	0.3	0.667	0.973222	55	14 13 7 12 24 1 20 30 6 2 21 17 10 4 8 23 22 16 15 25 19 28
	0.4	0.667	0.979658	55	21 14 8 20 12 16 13 2 27 24 26 7 17 3 23 6 30 10 19 1 18 4

	0.5	0.733	0.980945	54	1 2 28 14 12 8 13 21 6 10 3 11 17 7 16 23 30 24 19 26 18 4
	0.6	0.733	0.980945	54	1 2 28 14 12 8 13 21 6 10 3 11 17 7 16 23 30 24 19 26 18 4

Table 27. Optimal solutions for the network in figure 7, when T=11.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(120,80),11	0.3	0.667	0.979658	55	17 12 24 21 30 3 28 11 10 23 13 7 6 29 20 26 1 25 8 16 14 2
	0.4	0.600	0.981123	56	23 3 24 2 25 29 6 30 12 7 10 15 17 1 26 20 21 18 16 8 4 13
	0.5	0.733	0.980945	54	1 3 16 18 12 25 7 10 2 6 24 21 8 23 13 20 28 17 4 15 26 14
	0.6	0.667	0.982246	55	25 1 28 14 15 8 3 13 11 30 12 21 10 17 23 2 6 24 27 7 20 16

Table 28. Optimal solutions for the network in figure 7, when T=9.

(d_1, d_2, T)	α	$Best\ \mu_{\tilde{s}}(P)$	R	$S_l(p)$	P
(130,70),9	0.3	0.600	0.979645	56	21 20 14 28 2 23 24 4 1 6 30 8 7 12 3 27 10 22 25 19 13 16
	0.4	0.667	0.979658	55	18 2 16 30 14 1 8 25 26 24 4 13 7 11 20 10 6 15 3 21 17 12
	0.5	0.733	0.979658	54	12 14 2 20 19 16 17 30 26 24 23 7 18 3 21 6 13 10 25 1 28 4
	0.6	0.733	0.979658	54	2 14 8 20 21 16 15 30 17 13 24 7 19 28 23

					12 6 10 11 1 3 4

8. Conclusion

This paper presented an approach based on GA with fuzzy optimization to determine the optimal components that can be assigned for an SFN to maximize system reliability and minimize total lead-time. Our work concentrated on solving the optimal CAP problem for a two-commodity flow network via disjoint MPs. The suggested technique is examined on different examples four nodes, six nodes and fourteen nodes computer networks. The obtained solution is distinguished by the minimum total lead-time and the maximum reliability value.

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