Multi-objective components assignment problem for multi-source multi-sink flow networks

Noha Nasr Elden¹, M. R. Hassan¹ and M. H. Abd El-Aziz²

¹Mathematics and Computer Science Department, Faculty of Science, Aswan University, Aswan, Egypt
²Information systems Department, Faculty of Computer and Information Sciences, Ain Shams University, Egypt

Abstract — The multi-objective components assignment problem (MOCAP) for multi-source multi-sink flow networks when each component has an assignment cost is never discussed. The main objective of MOCAP is to search the optimal components that maximize network reliability of multi-source multi-sink flow networks and minimized the total assignment cost. An approach based random weighted genetic algorithm (RWGA) is proposed to solve the MOCAP. The Optimal Components Assignment Problem (OCAP) has a solution that is produced by RWGA. The results demonstrated that using the suggested method, optimal component assignment yields the greatest reliability, lowest assignment cost, and shortest total lead-time. The proposed algorithm has been applied to various networks to assert its efficiency in comparison with other approaches based on single genetic algorithm. We applied it to different types of network models, including two-source two-sink networks and three-source two-sink networks, with varying numbers of available components. Also, the obtained results show that the proposed RWGA approach works well and find optimal solutions for all studied cases.

Keywords: — components assignment problem, network reliability, Stochastic-flow networks, Genetic algorithm, multi-source multi-sink networks.

1. Introduction

Network reliability is defined as the possibility that a specified amount of flow can be successfully transmitted from source to destination via a stochastic-flow network (SFN) [1]. One of the key problems in production and service systems is the assignment problem (AP), which has drawn a lot of attention from scholars. The AP is in charge of allocating resources one to one to various activities. The generalized assignment problem is defined as the problem of determining the cheapest way to assign n jobs to m agents so that each job is assigned to exactly one agent, subject to an agent's capacity [2]. The components assignment problem (CAP) is a significant problem in the field of system reliability analysis; finding an optimal component assignment is critical to maximise system reliability and improve system performance [3]. Many researchers have studied CAP for an SFN using various algorithms to maximize network reliability under different constraints. Lin in [4] proposed an algorithm to generate all lower objective components assignment problem for multi-source two-sink flow networks and three-source two-sink networks, with varying numbers of available components. Also, the obtained results show that the proposed RWGA approach works well and find optimal solutions for all studied cases.

Keywords: — components assignment problem, network reliability, Stochastic-flow networks, Genetic algorithm, multi-source multi-sink networks.
time and assignment cost. In [12] authors proposed RWGA based approach to find the best solution characterized by the maximum reliability of the system with minimum total lead-time and assignment cost. In [13], the CAP was expressed as a fuzzy optimization problem, with node failure taken in to consideration [14]. In [15] described a MOPSO-based approach for selecting the best components to allocate to an SFN taking into consideration triple constraints; system reliability, total cost and total lead-time. Lin and Yeh investigated the CAP in terms of reliability and total cost [5] [6] and [16]. The goal was to maximize dependability while minimizing total assignment cost. In [17], Lin devised a technique to generate every lower boundary point in an SFN of multi-commodity demand in the presence of a budget constraint in order to evaluate system reliability, where each node and arc owns a large number of possible capacities. The problem of component assignment to maximize network reliability when each component has both an assignment cost and a lead-time is discussed in [18].

To maximise system stability in multi-source multi-sink SFN, authors in [19] examined the issue of distributing different resources at source nodes. In [20] given the transmission cost constraints, an algorithm was developed to solve it. Authors in [21] suggested modifying methods to solve the resource allocation problem for an uncertain multi-source multi-sink flow network, able to adjust for variations in resource demand or the attributes of the SFN. The multi-source multi-sink SFN flow assignment problem was discussed in [22] and [23]. In [22] discusses a double-resource assignment problem for maximizing computer network reliability. Transmission lines and transmission facilities are the two categories into which the resources are divided. Also in [23] determine the best resource flow allocation and control strategy for arcs and nodes. The multi-source multi-sink flow network system reliability optimization problem is defined as the search for optimal components that maximize reliability while minimizing total assignment cost. As a result, a genetic-based a method for resolving the component assignment problem under budget constraints is proposed [24]. In [25] researched the CAP in terms of capacity vector reliability within the constraints of an assignment budget. Using a Random Weighted Genetic Algorithm (RWGA), a solution to the Optimal Components Assignment Problem (OCAP) is given. The findings showed that optimal component assignment results in the maximum dependability, lowest assignment cost, and shortest total lead-time utilizing the proposed method [15].

Our concentration is mostly in this publication on optimizing the system reliability of multi-source multi-sink stochastic flow networks operating within a specified assignment budget. Our system reliability evaluation is based on the search for a group of lower boundary points, [24] and [26]. This study is different from [24] by solving the problem using multi objective GA algorithm. In contrast [24], search for the best components using single GA. The primary goal here is to find the optimal components that maximize system reliability while minimizing total assignment cost. In addition, an algorithm based on RWGA is presented to solve OCAP in multi - source multi- sink network.

The remainder of this paper is structured as follows: Section 2 discusses necessary notations, while Section 3 describes problem formulation. Section 4 then provides proposed algorithm. Section 5 explains experimental results. Section 6 presents comparison and discussion. Finally, Section 7 shows the conclusion.

2. Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Number of nodes.</td>
</tr>
<tr>
<td>A</td>
<td>Set of arcs.</td>
</tr>
<tr>
<td>S</td>
<td>Group of source nodes.</td>
</tr>
<tr>
<td>T</td>
<td>Group of sink nodes.</td>
</tr>
<tr>
<td>M</td>
<td>Group of sink SFN,</td>
</tr>
<tr>
<td>dw,j</td>
<td>The demand of resource w for sink node tj.</td>
</tr>
<tr>
<td>rw,j</td>
<td>The maximum resource w that a source node si can provide.</td>
</tr>
<tr>
<td>p</td>
<td>The population size.</td>
</tr>
<tr>
<td>g</td>
<td>The maximum generation.</td>
</tr>
<tr>
<td>cr</td>
<td>The rate of crossover.</td>
</tr>
<tr>
<td>mm</td>
<td>The rate of mutation.</td>
</tr>
</tbody>
</table>

3. Problem Formulation

Let us suppose $\mathcal{B} = \{\mathcal{b}_1, \mathcal{b}_2, \ldots, \mathcal{b}_n\}$ is set of available components given to set of arcs A. The total cost for the problem is $C(\mathcal{B}) = \sum_{c=1}^{m} c(\mathcal{b}_c)$ and the corresponding system reliability $R_s(\mathcal{B})$, evaluated using [24]. Then The OCAP mathematical programming formulation is as follows:

Maximize $R_s(\mathcal{B})$  \hspace{1cm} (1)

Minimize $C(\mathcal{B})$  \hspace{1cm} (2)
Subject to:
\[
\forall i \neq e \quad \text{for } i \neq e
\]

Since the multi-objective components assignments problem is converted into either a multi-objective minimization problem or a multi-objective maximizing problem in the case of a maximum and minimum goal [27] [28] [29]. The initial issue formulation can then be modified to be of the minimum form [18]:

\[
\text{Minimize } 1 - R_s(\beta) \quad \text{(4)}
\]

\[
\text{Minimize } C(\beta) \quad \text{(5)}
\]

4. The proposed Algorithm

The subsections that follow explain the steps of the proposed algorithm. To produce new offspring, we employ the modified uniform crossover and mutation described in [10].

4.1 Cross over

Following is a definition of the crossover operation: Given two parents, a new offspring is generated at random by choosing genes from each of them. Figure 1 shows how a crossover occurs.

<table>
<thead>
<tr>
<th>Parents</th>
<th>Offspring</th>
<th>Final offspring</th>
<th>Fig. 1 Modified Crossover</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5, 2, 1, 3, 4, 6)</td>
<td>(5 → 3, 1 → 2, 4, 5 → 6)</td>
<td>(5, 2, 1, 3, 4, 6)</td>
<td></td>
</tr>
<tr>
<td>(3, 1, 4, 2, 6, 5)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.2 Mutation

We note that the swap mutation is applied to prevent duplicate genes from existing in a genome chromosome. Figure 2 shows how a mutation occurs.

<table>
<thead>
<tr>
<th>Parents</th>
<th>Offspring</th>
<th>Final offspring</th>
<th>Fig. 2 Mutation process</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5, 3, 1, 2, 4, 6)</td>
<td>(5 → 2, 3 → 5, 4, 6)</td>
<td>(2, 3, 1, 5, 4, 6)</td>
<td></td>
</tr>
</tbody>
</table>

4.3 Fitness

Let \( R_s(\beta) \) and \( C(\beta) \) represent the corresponding values for the solution \( i \) where \( i = 1, 2, \ldots, P \):

Step 1: Determine the normalized values of \( R_s(\beta) \) and \( C(\beta) \) as follows:

Step 1.1. The normalized value of \( R_s(\beta) \):

\[
NR_s(\beta) = \frac{R_s(\beta)}{\text{Max}(R_s(\beta), R_s(\beta), \ldots, R_s(\beta))}
\]

Step 1.2. The normalized value of \( C(\beta) \):

\[
NC(\beta) = \frac{\text{Min}(C(\beta), C(\beta), \ldots, C(\beta))}{C(\beta)}
\]

Step 2: For each solution, compute the Fitness value as follows:
Step 2.1. For each objective k, generate a random number $u_k$ in the range $[0, 1]$, where $k = 1, 2, 3$.

Step 2.2. Determine the random weight of each objective $k$ as follows:

$$w_k = \frac{u_k}{\sum_{i=1}^{3} u_i}$$

Step 2.3. Evaluate the solution's fitness as:

$$f(i) = w_1 \cdot NR_s(i) + w_2 \cdot NC(i)$$

Step 3: Determine the probability of each solution being chosen.

$$P(i) = \frac{(f(i) - f_{\text{min}})}{\sum_{j \in \mathcal{P}} (f(j) - f_{\text{min}})}$$

where, $f_{\text{min}} = \min\{f(i), i \in \mathcal{P}\}$.

### 4.4 The algorithm

The whole algorithm used to solve the OCAP problem is described in the steps that follow:

1) Begin
2) Set $p, q, cr, mr$
3) Read network information
4) $gn = 0, gt = 0$
5) Create the initial population, which should include successful individual $\beta$
6) Evaluate initial population (Calculate $R_s(\beta), C(\beta)$ and $f(\beta)$)
7) While $gn < q$
8) While $gt < p$, do
9) Select two chromosomes using Roulette Wheel [24].
10) Apply crossover in accordance with $cr$.
11) Apply mutation in accordance with $mr$.
12) Calculate $R_s(\beta)$ and $C(\beta)$
13) $gt = gt + 1$
14) End do
15) Evaluate $f(gt)$, $gt = 1, 2, \ldots, p$
16) $gn = gn + 1$
17) End do
18) Report the optimal solutions
19) End.

### 5. Experimental Results

#### 5.1. Two-Source Two-Sink Network

Figure 3 depicts a computer network with two sources and two sinks as our first example. Available components are listed in Table 1, and the minimal paths $MP$s for this network are as follows:

$MP_{1,1,1} = \{a_1, a_5\}$, $MP_{1,1,2} = \{a_1, a_6, a_9\}$, $MP_{1,1,3} = \{a_2, a_7, a_9\}$, $MP_{1,2,1} = \{a_1, a_6, a_{14}\}$, $MP_{1,2,2} = \{a_2, a_7, a_{14}\}$, $MP_{2,1,1} = \{a_3, a_7, a_9\}$, $MP_{2,1,2}$
= \{a_4, a_8, a_{13}, a_9\} \), MP_{2,1} = \{a_3, a_7, a_{14}\}, \) MP_{2,2} = \{a_4, a_8, a_{13}, a_{14}\}, \) MP_{2,3} = \{a_4, a_8, a_{10}\} \) and \) MP_{2,4} = \{a_4, a_{11}, a_{12}\} \). 

We suppose that \( R = (r_{1,1}, r_{1,2}, r_{2,1}, r_{2,2}) = (15,17,10,13) \), \( D = (d_{1,1},d_{1,2},d_{2,1},d_{2,2}) = (9,10,5,8) \), and \( (41,52,51,32,61,52,31,42,21, 62,51,52,41,22,31,22,11,32,51,51) \) are costs of the available components. Table 2 lists the values of fitness function, \( R_0(\beta) \) and \( C(\beta) \) for the first example and figure 4 shows the fitness values for this network in figure 3.

![Figure 3 Network with two sources and two sinks](image)

**Table 1** Available components.

<table>
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<tr>
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<th>0</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<th>8</th>
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<td>0.001</td>
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<td>0.004</td>
<td>0.005</td>
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<td>0.015</td>
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<td>0.012</td>
<td>0.015</td>
<td>0.017</td>
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<td>0.891</td>
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</table>
Table.2  Obtained results for example 5.1.

<table>
<thead>
<tr>
<th>NO.</th>
<th>f(ß)</th>
<th>R_s(ß)</th>
<th>C(ß)</th>
<th>Assigned Components (ß)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.973529</td>
<td>0.875641</td>
<td>170</td>
<td>4 11 8 10 12 2 4 18 6 20 5 1 10 18</td>
</tr>
<tr>
<td>2</td>
<td>0.978824</td>
<td>0.956305</td>
<td>170</td>
<td>4 11 8 4 12 2 10 18 6 20 5 11 8 10</td>
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<td>3</td>
<td>0.971123</td>
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<td>0.989412</td>
<td>0.953119</td>
<td>170</td>
<td>4 7 10 18 2 3 5 11 8 16 6 11 8 10</td>
</tr>
</tbody>
</table>
5.2. Two-Sink Network

Figure 5 depicts the second example in this work, which includes three sources and two sinks. Table 3 shows the available components with capacities, costs, and probabilities. The network has the following minimal paths: \( MP_{1,1,1} = \{a_1, a_7\}, MP_{1,1,2} = \{a_2, a_9\}, MP_{1,2,1} = \{a_1, a_9\}, MP_{2,1,1} = \{a_3, a_8\}, MP_{2,2,1} = \{a_4, a_{10}\}, MP_{3,1,1} = \{a_5, a_9\}, \) and \( MP_{3,2,1} = \{a_6, a_{10}\}. \) (3 7 8 12 11 2 9 1 10 6) is the values of \( C. \) \( R = (r_{1,1,1}, r_{1,2,2}, r_{1,3}, r_{2,1,2}, r_{2,2,2}, r_{2,3}, r_{3,1}, r_{3,2}, r_{3,3}) = (5, 2, 3, 5, 3, 2, 2, 3) \) and \( D = (d_{1,1,1}, d_{1,2,2}, d_{2,1,2}, d_{2,2,2}, d_{3,1,1}, d_{3,2}) = (3, 1, 2, 2, 1, 3). \)

5.2. Two-Sink Network

Fig. 4 The fitness values for example 5.1.
Fig. 5  Network with three sources and two sinks.

Table.3  Available components with capacities, costs, and probabilities.

Table.4  Obtained results for example 5.2.
Fig. 6 The fitness values for example 5.2
6. Discussion and Comparison

This study presents and solves the optimal component assignment in the presence of two competing objectives: system reliability and cost. In this paper, we present a new approach based on multi RWGA for solving multi objective component assignment problems with two constraints: network reliability and total cost. Using the proposed approach on a network to compare the results with the other obtained by approach based on single GA proposed by Elden et al. [24]. Table 2 and 4 show the results obtained by the proposed algorithm applied in example 5.1 and 5.2 respectively.

<table>
<thead>
<tr>
<th>Studied Examples</th>
<th>Elden et al. [24]</th>
<th>RWGA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R_s(C)</td>
<td>Cost</td>
</tr>
<tr>
<td>Example 1</td>
<td>0.981306</td>
<td>170</td>
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<tr>
<td>Example 2</td>
<td>0.540484</td>
<td>21</td>
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</table>

The results of applying the suggested algorithm in comparison with Elden et al. [24] are shown in Table 5. It is discovered that the proposed approach's values of system reliability are better than those obtained by Elden et al. [24]; in addition the Costs are equal. As a result, the proposed method yields more optimum solutions.

Conclusion

Our work investigated how to find the best maximum assignment component for SFN system dependability with a minimum of assignment cost. A multi-objective component assignments problem is explained and expressed as a multi-objective minimization problem, with system reliability and assignment cost as constraints. In addition, a multi-objective to solve the problem, a GA-based RWGA strategy is suggested. The presented issue using the proposed method, the system achieves the optimal solution. The reliability is maximized, as is the assignment cost is kept to a minimum.

References


